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Marchisotto, Elena Anne Corie (1-CASN);

Rodríguez-Consuegra, Francisco [Rodríguez Consuegra, Francisco A.]

(E-VLNCQD-LQ);

Smith, James T. (1-SFSU-MS)

★The legacy of Mario Pieri in foundations and philosophy of mathematics.

With a foreword by David E. Rowe.

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Continuing previous work of two of the present authors [E. A. C. Marchisotto and J. T. Smith, *The legacy of Mario Pieri in geometry and arithmetic*, Birkhäuser Boston, Boston, MA, 2007; MR2310967], this book gives a very thorough and carefully produced account of Pieri's foundational work in geometry, including some 200 pages of translations of his works. Pieri probably deserves to be better known next to bigger names in logicism such as Peano, Hilbert, and Russell. His work on formal synthetic geometry was in many ways a natural culmination of 19th-century trends in a logicising direction, but, as synthetic geometry all but died as a research field in the 20th century, few in subsequent generations were in a natural position to celebrate these achievements.

At the first International Congress of Philosophy in Paris in 1900, Pieri gave an address titled *On geometry envisaged as a purely logical system*, translated in full here, in which he outlines many now-familiar points about the formalising program as well as gives a taste of his own technical work in this area. This congress was formative to Russell, and the authors go on to consider the relation between Pieri and Russell in more detail, including Russell's notes taken while reading Pieri and a brief letter from Pieri to Russell.

Also translated here is Pieri's treatise *The Principles of the Geometry of Position*, on projective geometry. In the mid-19th century, von Staudt had famously developed projective geometry without reference to metric notions. In Pieri's time this work was celebrated as a precursor of the formal approach to geometry—indeed, one indicator of this high esteem is that Pieri translated von Staudt's treatise into Italian. Nevertheless, Pieri takes the ideal of purity of method further still. In particular, he builds up projective geometry from only two primitive concepts—point and join—whereas von Staudt for example had treated the simple angle space (einfacher Winkelraum; a portion of a bundle of rays) as a primitive concept.

A similar type of purity is achieved in Pieri's *Monograph on points and on motion*, also translated here, in which he develops the geometry of three-dimensional Euclidean space using only two primitives: point and motion (direct isometry). In particular, the concept of line can be introduced via the definition that three points are collinear if and only if there exists a proper motion that leaves these three points fixed. Even Peano, whose school Pieri can be said to belong to, had taken line segment as an additional primitive notion in his formalisation of geometry.

Viktor Blåsjö